AN ANALYTICAL INVESTIGATION OF THE EFFECT OF A SINUSOIDAL INPUT ON THE RESPONSE CHARACTERISTICS OF A TYPICAL BARO SENSING SYSTEM

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Approved:

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NOTATIONS AND ABBREVIATIONS

Latin

A_o	orifice area, sq. ft.
A	amplitude of input pressure, psi
a	amplitude of response pressure, psi
B	System Constant ($B = \frac{K\,R}{g} \quad \frac{A_o}{V} \quad \frac{T_R}{T}$) , 1/sec.
c	speed of sound, fps
$\frac{dm}{dt}$	mass flow per unit time, slugs per sec
D_p	pipe diameter, in.
d	orifice diameter, in.
g	acceleration of gravity; 32.17 ft. per sec. per sec.
K	discharge coefficient
M	Mach number, $\frac{U}{c}$
P_1	pressure upstream to test orifice, psia
P_2	pressure downstream of test orifice, psia
p	response pressure, psi
p_i	input pressure, psi
r	pressure ratio, P_1/P_2 ; p/p_i
R	gas constant; 53.3 for air

^{o}R	degrees Rankine
RN	Reynolds number, $\frac{\rho\, U\, d}{\mu}$
T	temperature ahead of test orifice; input temperature, deg. R
T_R	temperature of the Baro Sensing System, deg. R
t	time, sec.
U	velocity, fps
V	chamber volume of the Baro Sensing System, cu. in.
w	weight flow per unit time, lbs. per sec

Greek

β	ratio of the orifice diameter to the pipe diameter
γ	ratio of specific heats; 1.4 for air
μ	coefficient of viscosity, lbs.-sec./sq. ft.
Ξ	$= \frac{w \sqrt{T}}{A_o}$, $\frac{\text{lb. } (^{o}R)^{1/2}}{\text{sec. in.}}$
τ	response period, sec per cycle
ϕ	phase shift, deg.
Ω	$= \frac{w \sqrt{P}}{A_o\ P_1}$; $(^{o}R)^{1/2}$/sec.

Abbreviations

psi	pounds per square inch

psia	pounds per square inch absolute
lb(s)	pound(s)
in.	inch(es)
ft.	foot - feet
fps	feet per second
sec	second(s)
cu. in.	cubic inches

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SUMMARY

This paper presents an analytical investigation of the effect of a sinusoidal input on the response characteristics of a typical Baro Sensing system.

A nonlinear theory which is based on an empirical equation describing the mass flow through a sharp-edged orifice in terms of pressure ratio across the orifice is developed. The resulting nonlinear differential equation relates the response pressure to the input pressure. This nonlinear differential equation is solved by means of an analog computer and the results are checked by means of a digital computer.

The problem is solved for a range of values of Mach number, input amplitude, and System Constant at an altitude of 5,000 feet.

The most important results are: the response pressure varies periodically, with a period slightly different from that of the input pressure; the response amplitude is smaller than that of the input function, and there is a phase shift between the input pressure and the response pressure.

CHAPTER I

INTRODUCTION

During the past decade the large increase in velocities of test vehicles, airplanes, and guided missiles has presented the necessity of predicting the response experienced by the Baro Sensing system in these vehicles during diving, climbing, maneuvering, or accelerating at high speeds.

This report is an analytical investigation of the effect of a sinusoidal pressure input, which is similar to the input applied to an instrument installed in an airplane or missile that is performing an oscillatory motion, on the response characteristics of the Baro Sensing system of the aircraft in question.

The case of a step input and a linear input, which is similar to the input applied to an instrument installed in an aircraft which is either diving or climbing, has been solved by Vaughn (1).

The author in solving the case of a sinusoidal input used a nonlinear quasi-empirical theory.

CHAPTER II

THEORY

The development of this nonlinear theory is based on an empirical equation which describes the mass flow through a sharp-edged orifice in terms of the pressure ratio across the orifice. This empirical relation was based on and developed from tests conducted by Perry (2).

To begin a description of the tests it will be necessary to explain the two flow factors that are used by Perry, Ξ and Ω.

Flow through an orifice is found to be a function of the flow area, A_o , the Reynolds number, RN, the diameter ratio, β , the pipe diameter, D_p , the specific heat ratio, γ , the head pressure, P_1 , the back pressure, P_2 , and the absolute temperature, T ;

thus

$$w = f\,(A_o\,,\ RN,\ \beta\,,\ D_p\,,\gamma\,,\ P_1\,,\ P_2\,,\ T)$$

The effect of RN becomes negligible above RN = 100,000 and all tests were run above this number. At very small diameter ratios the effect of the diameter ratio and the size of pipe tend to become negligible. In the development of the flow rate equation (reference 2) it is shown that the flow rate varies inversily as the square root of the absolute temperature. Incorporating the above assumption the flow rate is written as,

$$w = \frac{A_o}{\sqrt{T}} \; f\,(P_1, P_2)$$

Then, grouping w, A_o , and T gives

$$\frac{w\sqrt{T}}{A_o} = f\,(P_1, P_2)$$

This factor will be called Ξ . So $\Xi = w\sqrt{T}/A_o$ and is seen to be a function of the absolute pressures.

Now suppose that is divided by P_1 , then

$$\frac{\Xi}{P_1} = \frac{w\sqrt{T}}{P_1\;A_o}$$

This factor will be called Ω and $\Omega = w\sqrt{T}/P_1 A_1$; Ω is seen to be a function of the pressure ratio alone.

The significance of Ω will be seen more clearly by considering the equation representing an adiabatic expansion through an ideal nozzle

$$w = \frac{P_1\;A_o}{\sqrt{T}} \sqrt{\frac{2\,g}{R} \quad \frac{\gamma}{\gamma - 1}} \quad r^{1/\gamma}\sqrt{1 - r^{\frac{\gamma - 1}{\gamma}}} \qquad (1)$$

where r is the pressure ratio P_2/P_1 .

Subsequently

$$\Omega = \sqrt{\frac{2g}{R} \quad \frac{\gamma}{\gamma - 1}} \; r^{1/\gamma} \sqrt{1 - r^{\frac{\gamma - 1}{\gamma}}} \qquad (1a)$$

All tests were run holding a constant head pressure and varying the back pressure. Values of Ξ were calculated for each run. A plot of Ξ versus P_2 appears in Fig. 3 of reference 2.

Of themselves these curves are of little value. However, the next step was to divide each of these curves by the parameter, P_1. The interesting result is that all of those curves now seem to lie on the same line as is seen in Fig. 4 of reference 2. This is a plot of Ω versus r. The values of Ω were obtained from the "smoothed" data taken from Fig. 3 (reference 2). As in the nozzle flow equation, Ω appears to be a function of r only.

As may be noted from Fig. 4 (reference 2), it would be rather difficult to draw an accurate line through the lower pressure ratio section (choked flow regime) because of the flatness of the curve. To study the characteristics of orifice flow more closely, Ω versus r was plotted on elliptical coordinates, i.e. all values of Ω and r were squared.

The orifice characteristic is shown on elliptical coordinates in Fig. 5 (reference 2). Some interesting conclusions can be drawn from this plot. First it is seen that there is a fundamental difference in the nature of the flow in the subcritical region ($1.0 \geq r \gtrsim .528$) and in the critical region ($.528 \gtrsim r \geq 0$).

In the subcritical region the points are found to lie on a straight line; and this of course represents an ellipse. In the critical region, the flow characteristic falls away from an ellipse and appears to be an entirely different function. It is now a simple matter to express the flow relationship in the subcritical region by an equation of an ellipse

$$\Omega = 0.465 \sqrt{1 - r^2}$$

or in terms of w

$$w = 0.465 \frac{A_o}{\sqrt{T}} \sqrt{P_1^2 - P_2^2}$$

but since $$w = \frac{dm}{dt} g$$

$$\frac{dm}{dt} = \frac{0.465 A_o}{g \sqrt{T}} \sqrt{P_1^2 - P_2^2} \qquad (2)$$

where P_1 and P_2 are respectively the steady state upstream and downstream pressures across the orifice and T is the upstream temperature. The pressures in equation (2) can be time-dependent, if it is assumed that the delay in response of the mass flow to changes in pressure differential are insignificant. This, of course, means that it is assumed that the effect of the inertia of the air and the effect of the speed of pressure propagation in the plumbing is accountable. This is believed to be a reasonable assumption as long as the plumbing lines are not extremely long. Then, with a change in symbolism, equation (2) becomes

$$\frac{dm}{dt} = \pm \frac{K A_o}{g \sqrt{T}} \sqrt{\left| p_i^2 - p^2 \right|} \qquad (3)$$

where K is a constant for a particular type of plumbing and a given Reynolds number, p_i the input pressure and p the response pressure. The positive and negative signs are inserted to determine the direction of the flow (i.e. if $p_i^2 \geq p^2$ $\frac{dm}{dt} \geq 0$, and if $p_i^2 < p^2$ then $\frac{dm}{dt} < 0$).

Now, the equation of state of the air inside the response chamber is,

$$p = \frac{m R T_R}{V} \qquad (4)$$

where V is the volume and T_R the temperature of the response chamber.

Since p is a function of time, and R and V are not, the derivative of equation (4) is

$$\frac{dp}{dt} = \frac{R T_R}{V} \frac{dm}{dt} + m \frac{R}{V} \frac{dT_R}{dt}$$

Fortunately, the last term in equation (5) is insignificant with respect to the first term, mainly because T_R is almost constant with time. This assumption is justified for most systems since the ratio of surface area to volume is very large, thus resulting in a large heat transfer rate to the interior of the vehicle which acts as a heat sink. The resulting instrument temperature essentially assumes the temperature of its surroundings. Hence it is assumed that the rate derivative of T_R can be neglected, thus

$$\frac{dp}{dt} = \frac{R\,T_R}{V}\,\frac{dm}{dt} \tag{6}$$

Substituting equation (6) into equation (3) and solving for $\frac{dp}{dt}$,

$$\frac{dp}{dt} = \pm\; B\,\sqrt{\left|p_i^2 - p^2\right|} \tag{7}$$

where

$$B = \frac{K\,R}{g}\,\frac{A_o}{V}\,\frac{T_R}{\sqrt{T}} \tag{8}$$

The System Constant, B , is seen to be a function of the system geometry, A_o/V, the upstream temperature, T , and the discharge coefficient, K , which is a function of pressure ratio only (3 and 4) for RN above 10,000. The constant B is termed a lump constant since it implicitly accounts for the variations mentioned above. Equation (7) is the basic differential equation relating the response of the internal pressure p to the applied pressure p_i and can be solved for any time dependent pressure input. It has been solved for a step pressure input, which is useful in determining the System Constant B . It has also been solved for an input pressure that varies linearly with time. This condition simulates the input applied to an instrument installed in an airplane or missile that is either diving or climbing. Both of the above solutions can be found in reference (1).

CHAPTER III

SOLUTION

The purpose of this report is the solution of equation (7) for a sinusoidal pressure input, which is similar to the input applied to an instrument installed in an airplane or missile that is performing an oscillatory motion about a level path. This level path was chosen to be 5,000 feet and the range of Mach numbers from 1.5 to 5.0.

The sinusoids considered were of the form

$$p_i = p_o + \frac{A}{2} \cos \omega t$$

where p_o is the ambient pressure. Values of A from one to five psi and the $\underline{\omega}$'s were found from the different values of A and M in the following manner:

$$p_i = p_o + \frac{A}{2} \cos \omega t$$

$$\frac{dp_i}{dt} = - \frac{A}{2} \omega \sin \omega t$$

and

$$\left(\frac{dp_i}{dt}\right)_{\omega t = \frac{\pi}{2}} = - \frac{A}{2} \omega$$

where $(dp_i/dt)_{\omega t} = \frac{\pi}{2}$ is the change of the input pressure with respect to time at 5,000 feet.

$$\frac{dp_i}{dt} = \frac{\partial p_i}{\partial x}\frac{dx}{dt} = \frac{\partial p_i}{\partial x} U = \frac{\partial p_i}{\partial x} (M\ c)$$

where c (speed of sound) is 1096.88 fps at 5,000 feet and $M = \frac{U}{c}$ is the Mach number, U being the vehicle velocity. By plotting altitude versus pressure the value of $\partial p_i / \partial x$ was obtained at 5,000 feet. This value was found to be -0.066 $lbs/ft.^3$.

Now since $(dp_i/dt)\frac{\pi}{2} = -\frac{A}{2}\omega$

and

$$(dp_i/dt)\frac{\pi}{2} = -0.066\ (1096.88)\ M$$

one can write an expression for ω in terms of A and M or:

$$\omega = \frac{2(72.39)\ M}{144\ A}\ 1/\text{sec} = (1.005\ \frac{M}{A})\ 1/\text{sec} \tag{9}$$

The values of $\underline{\omega}$ vary from about 0.1 to 2.5 per second (see Table 1).

Since the System Constant depends on Reynolds number and the type of plumbing as well as the type of the pressure measuring device installed in the vehicle, the effect of B was considered by taking a range of values of B between 0.2 and 7.0.

Equation (7) was solved by means of an analog computer for a number of combinations of the different parameters (A, ω , B), and the results were checked by means of a digital computer.

The programming and the block diagram, Fig. 1, were worked out by the Analog Computer Laboratory at the Engineering Experiment Station at the Georgia Institute of Technology.

Programming.--The final program for the sinusoidal response used the following components:

28 Operational Amplifiers

2 Electronic Multipliers

12 Coefficient Potentiometers

A block diagram schematically showing the component alignment is given in Fig. 1.

In the integration of the term

$$(p_i^2 - p^2) \tag{10}$$

the quantity was factored into

$$(p_i + p) (p_i - p) \tag{11}$$

and then the two factors multiplied. This operation saved one multiplier. A series diode V-circuit was used to take the absolute value of quantity (10) and a conventional square-root circuit followed to generate $\frac{1}{B} \frac{dp}{dt}$. A special Decision circuit was used to choose the proper sign on $\frac{dp}{dt}$. An integrator then generated p . The initial condition on p was chosen to be slightly less than $p_i(0)$, since it was desirable to have as little step transient as possible consistent with a stable square-root circuit.

Since the driving function and the response represented a relatively small signal riding on the ambient pressure, and since the "a - c" only was of interest, a biased arrangement was used to cancel out a part of the "d - c". This arrangement (see Fig. 1) uses amplifiers 18 and 31 for the response and 12 and 19 for the driving function.

In some cases a Delayed Sweep Circuit was employed to record the steady state solution only.

<u>Discussion of errors</u>.--The three principal sources of error are the Square-root circuit, the Decision circuit, and the Delayed Sweep circuit. The Square-root circuit introduces about 0.2 per cent error; the Decision circuit, about 0.1 per cent error; and the Delayed Sweep, about 0.03 per cent error. Other normal computing errors should not make the possible overall error greater than 3.0 per cent. These percentage errors are based on a full-scale voltage of 200 volts. With the scale factor of five volts per psi, the possible overall error should not exceed 1.2 psi. The probable error though is only about 0.25 per cent or in terms of psi, about 0.1 psi.

<u>Runs</u>.--Runs were made for various values of A, ω, and B, and are summarized in Table 1.

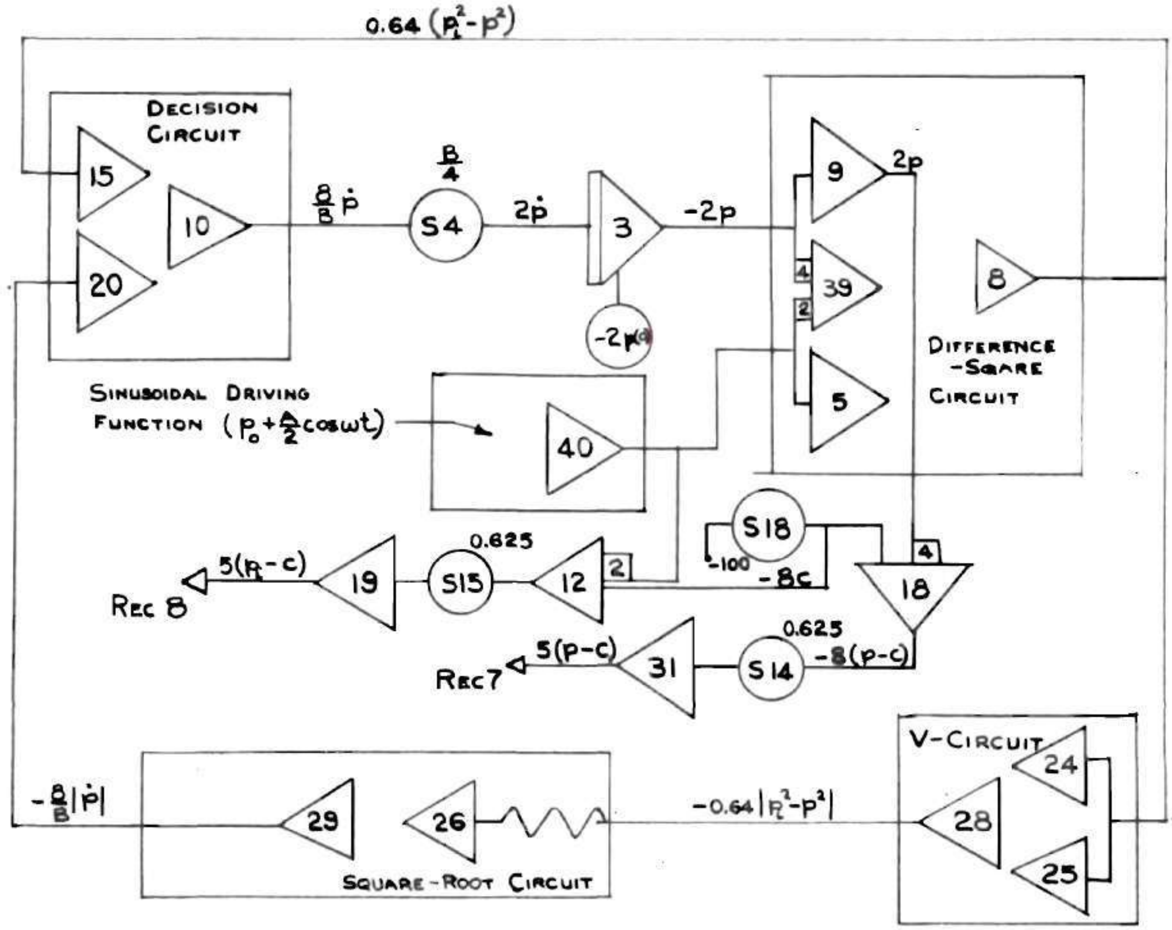
Representative runs are shown in Figs. 2 and 3.

The solution of equation (7) was checked by means of a digital computer at the Engineering Experiment Station at the Georgia Institute of Technology, and a comparison of the two solutions is given in Fig. 4.

Table 1

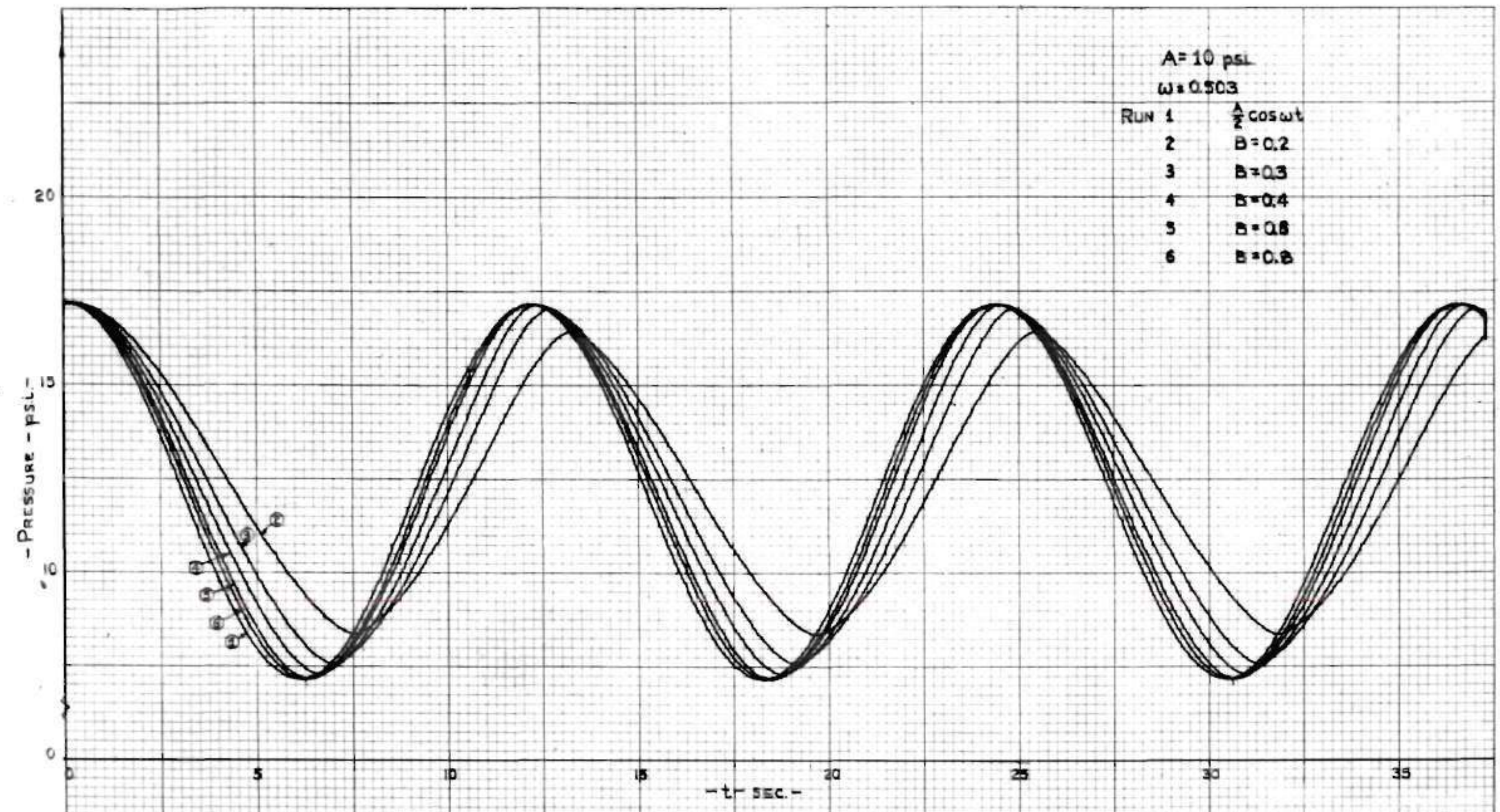
Runs

Sheet	A Input Amplitude, psi	in^1/sec	B System Constant in^1/sec					
1	5	0.5030	0.2,	0.3,	0.4,	0.6,	0.8	
2	5	0.4520	0.2,	0.3,	0.4,	0.6,	0.8	
3	5	0.3520	0.2,	0.3,	0.4,	0.6,	0.8	
4	5	0.2510	0.1,	0.2,	0.3,	0.4		
5	5	0.1510	0.1,	0.12,	0.16,	0.2,	0.3	
6	4	0.6280	0.2,	0.3,	0.4,	0.6,	0.8	
7	4	0.5660	0.2,	0.3,	0.4,	0.6,	0.8	
8	4	0.4400	0.1,	0.2,	0.3,	0.4,	0.6,	0.8
9	4	0.3140	0.1,	0.14,	0.2,	0.3,	0.4	
10	4	0.1890	0.1,	0.14,	0.2,	0.3		
11	3	0.8370	0.1,	0.2,	0.3,	0.4,	0.6,	0.8
12	3	0.7530	0.1,	0.2,	0.3,	0.4,	0.6,	0.8
13	3	0.5855	0.1,	0.2,	0.3,	0.4,	0.6,	0.8
14	3	0.4179	0.1,	0.2,	0.3,	0.4,	0.6,	0.8
15	3	0.2510	0.1,	0.14,	0.2,	0.3,	0.4	
16	2	1.2568	0.1,	0.2,	0.3,	0.4,	0.6,	0.8
17	2	1.1311	0.1,	0.2,	0.3,	0.4,	0.6,	0.8
18	2	0.8798	0.1,	0.2,	0.3,	0.4,	0.6,	0.8
19	2	0.6284	0.1,	0.14,	0.2,	0.3,	0.4,	0.6
20	2	0.3770	0.1,	0.14,	0.2,	0.3,	0.4	
21	1	2.5135	0.2,	0.3,	0.4,	0.6,	0.8,	1.0
22	1	2.2622	0.2,	0.3,	0.4,	0.6,	0.8,	1.0
23	1	1.7595	0.1,	0.2,	0.3,	0.4,	0.6,	0.8
24	1	1.2568	0.1,	0.14,	0.2,	0.3,	0.4,	0.6
25	1	0.7541	0.1,	0.14,	0.2,	0.3,	0.4,	0.6



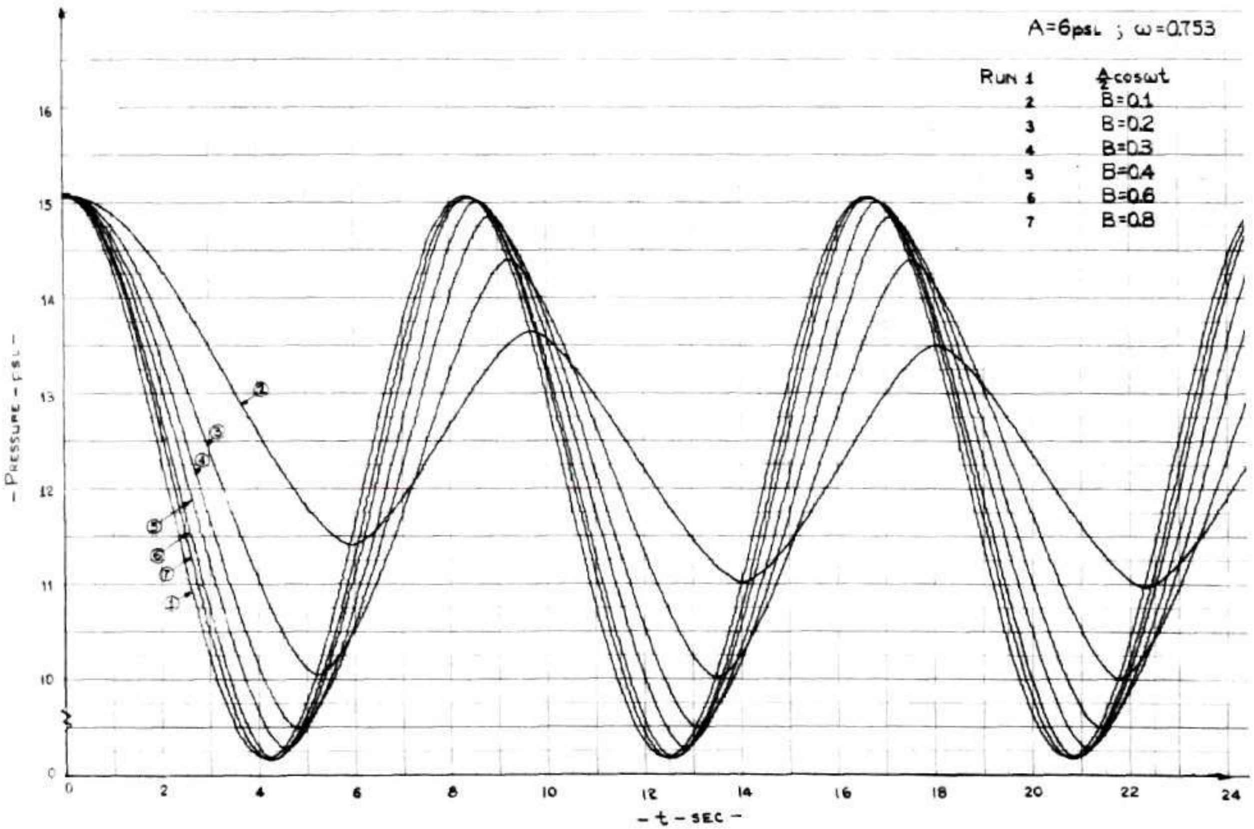
Program

Figure 1



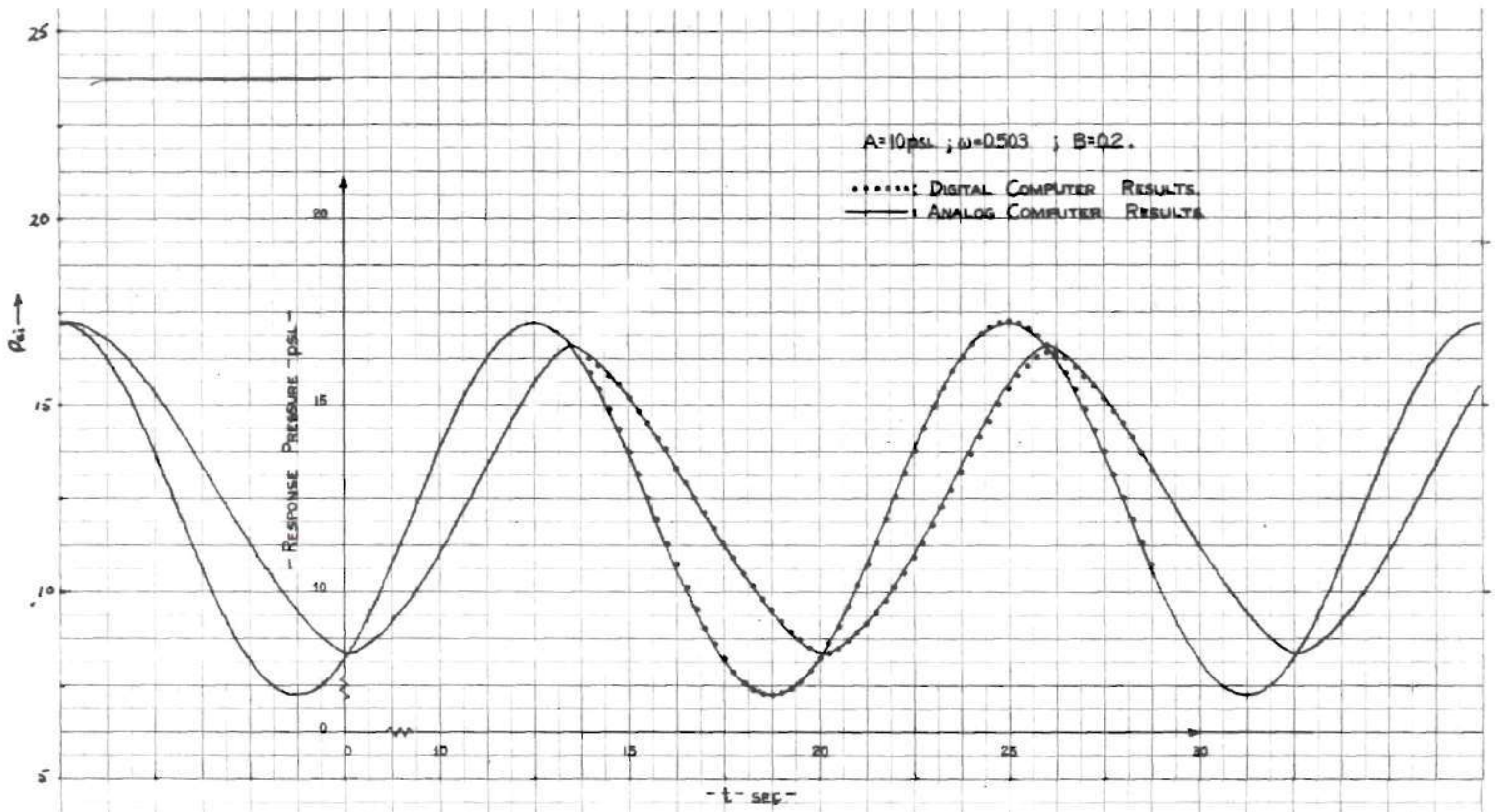
Pressure versus Time

Figure 2



PRESSURE VERSUS TIME

FIGURE 3



Response Pressure versus Time

Figure 4

CHAPTER IV

DISCUSSION OF RESULTS

It was observed from the steady state part of the solution that there is a phase shift between the input and the response pressures and also a distinct difference in amplitude (see Figs. 2, 3, and 4).

The phase shift, ϕ , and the response amplitude, a , as well as the response period, τ , are seen to be functions of the parameters A, ω , M , and B . The values of phase shift, response amplitude, and response period are tabulated in Table 2, and plotted in Figs. 5 - 9, 9 - 14, and 15 respectively.

Each curve that bears the number one in Figs. 5 through 14 corresponds to the same Mach number (M = 5). Similarly curves two correspond to M = 4.5, curves three to M = 3.5, curves four to M = 2.5, and curves five to M = 1.5.

The curves in each figure seem to form a family of curves.

Phase Shift.--It is observed from Figs. 5 through 9 that the smaller the B the larger the phase shift. This phase shift angle was measured on the p_0 - axis of the solution sheets, and it can be seen from Figs. 2, 3, and 4 that the phase shift is smaller at the top part of the response pressure curve than at the bottom. This occurs mainly because of the nonlinearity of the system.

The phase shift decreases as A increases keeping M and B constant.

Response Amplitude.--The response amplitude is never greater than the input amplitude. It approaches A as B increases. The effect of the System Constant is very small beyond the value of one, independently of A.

It can be noticed from Figs. 10 through 14 that the smaller the Mach number the smaller the effect of B on the response amplitude.

Response Period.--The response period does not change appreciably with B and it is slightly different from the input period (see Figs. 2 and 3), but it does change with A and M. As M increases, holding A constant, the response period decreases. On the other hand, holding M constant, the response period increases as A increases (see Fig. 15).

Table 2 a

Response Characteristics

	(input frequency) per. sec.	Response Period sec.	ϕ (Phase Shift in deg) B=0.2	B=0.3	B=0.4	B=0.6	B=0.8	Response Amplitude psi B=0.2	B=0.3	B=0.4	B=0.6	B=0.8
	2.5135	2.38	68.0	59.0	45.4	28.7	21.9	.84	1.20	1.49	1.80	1.93
	2.2622	2.60	66.0	54.0	42.8	26.3	19.3	.95	1.32	1.60	1.88	1.96
A = 2 psi	1.7595	3.52	57.5	42.8	28.2	18.5	13.7	1.16	1.54	1.78	1.94	2.00^{-}
	1.2568	4.80	43.8	31.1	21.8	12.9	8.7	1.39	1.80	1.87	1.98	--**
	.7541	8.30	26.9	16.7	11.3	5.2	--*	1.80	1.90	1.94	2.00^{-}	--
	1.2568	5.05	54.2	42.0	30.0	18.7	11.7	2.35	3.13	3.59	3.90	3.95
	1.1311	5.52	54.5	37.2	27.2	15.6	9.1	2.55	3.31	3.71	3.94	3.97
A = 4 psi	0.8798	6.84	48.4	30.0	20.2	11.3	6.8	3.04	3.67	3.89	3.97	4.00^{-}
	0.6284	10.20	29.2	18.0	11.5	5.7	---	3.52	3.85	3.94	4.00^{-}	---
	0.3770	17.00	22.7	8.0	7.2	---	---	3.89	3.92	4.00^{-}	---	---

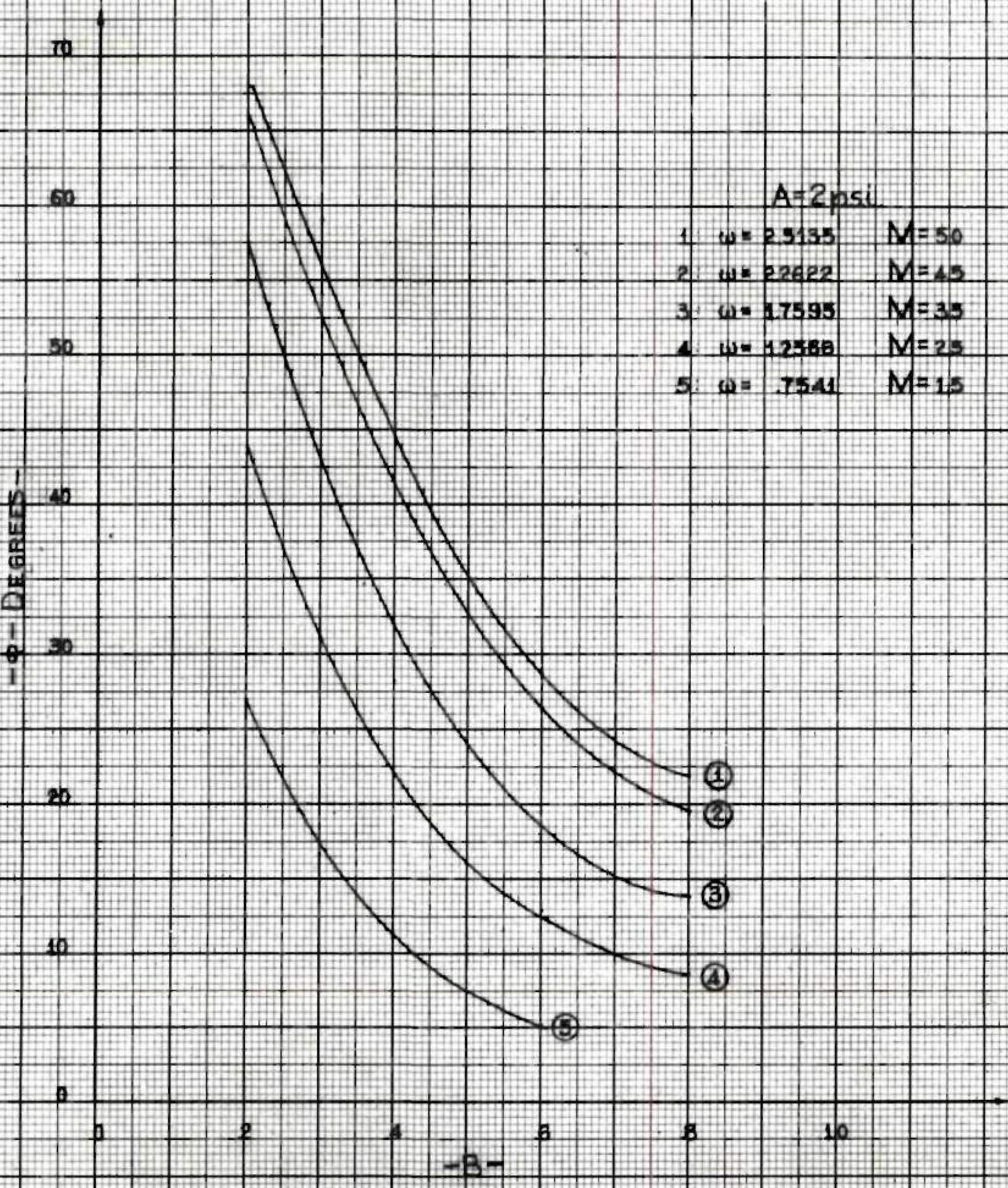
* No value obtained

** These values are equal to the corresponding A

Table 2 b

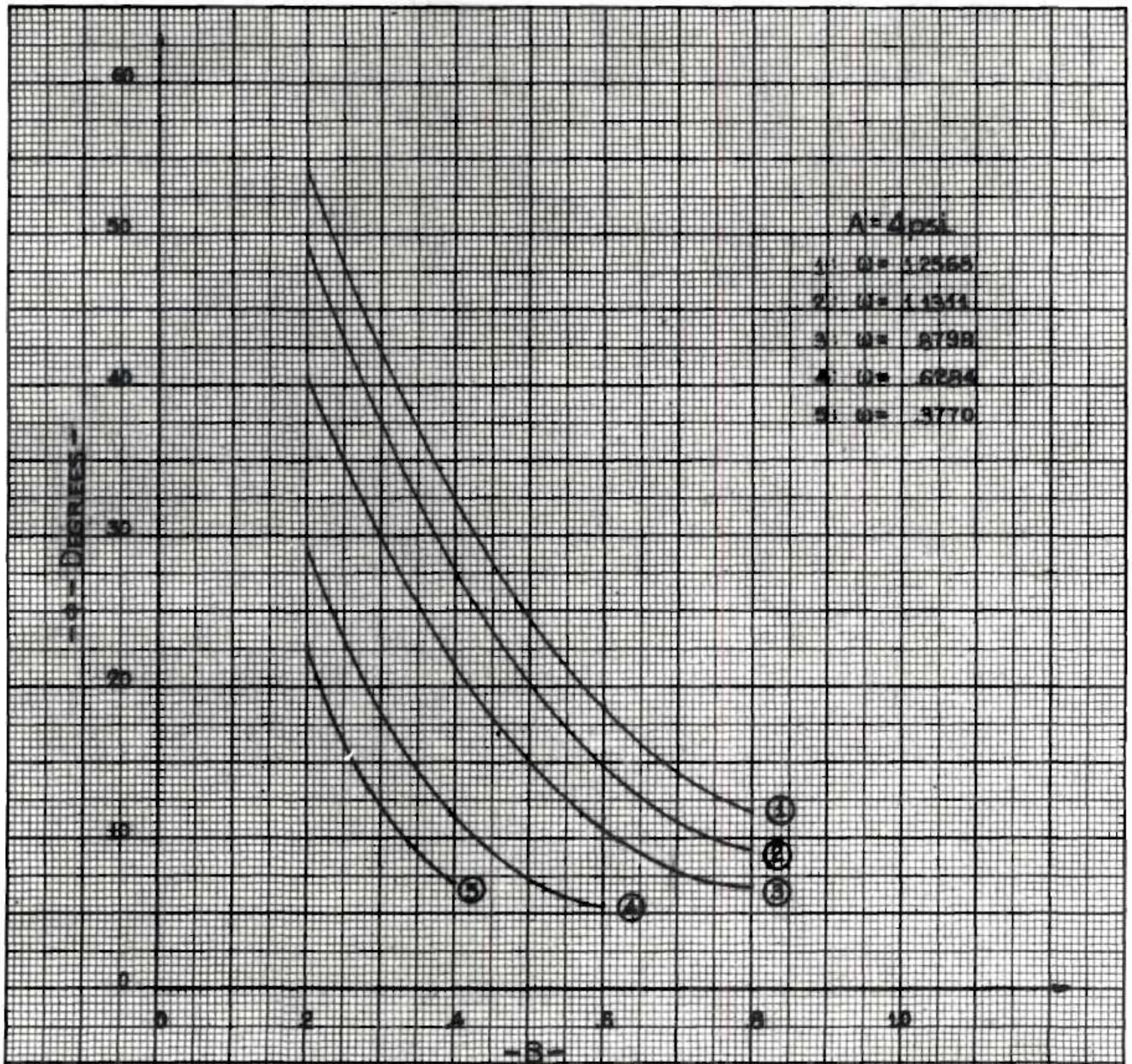
Response Characteristics

	(input frequency) per. sec.	Response Period sec.	ϕ (Phase Shift in deg) B=0.2	B=0.3	B=0.4	B=0.6	B=0.8	Response Amplitude psi B=0.2	B=0.3	B=0.4	B=0.6	B=0.8
	0.8370	7.55	47.0	31.5	21.9	11.9	7.6	4.08	5.12	5.63	5.85	5.90
	0.7530	8.30	45.0	30.4	19.1	9.1	5.2	4.35	5.35	5.72	5.89	5.97
A = 6 psi	0.5855	10.70	35.7	21.2	12.8	7.4	---	5.10	5.75	5.90	5.94	6.00^{-}
	0.4179	15.00	24.0	14.2	8.8	4.1	---	5.70	5.92	5.99	5.99	---
	0.2510	24.75	18.5	5.8	3.6	---	---	5.85	5.99	6.00^{-}	---	---
	0.6280	10.10	44.5	30.3	20.5	10.7	5.3	6.23	7.50	7.90	7.99	8.0^{-}
	0.5660	11.40	39.5	24.1	14.1	7.9	4.6	6.63	7.75	7.96	8.00	---
A = 8 psi	0.4400	14.85	34.4	20.4	12.5	7.0	4.0	7.14	7.82	8.00^{-}	---	---
	0.3140	19.50	19.0	9.3	5.5	---	---	7.75	7.95	---	---	---
	0.1890	32.50	8.3	4.4	---	---	---	8.00^{-}	8.00^{-}	---	---	---
	0.5030	12.20	38.4	23.6	14.8	8.1	5.2	8.08	9.45	9.82	9.99	10.00^{-}
	0.4520	13.50	35.4	20.0	13.3	6.7	4.6	8.50	9.65	9.95	10.00^{-}	---
A =10 psi	0.3520	17.30	24.5	13.5	9.9	3.6	2.6	9.33	9.90	9.99	---	---
	0.2510	24.25	14.8	7.4	5.6	---	---	9.81	9.99	10.00^{-}	---	---
	0.1510	43.50	6.5	3.3	---	---	---	9.88	10.00^{-}	---	---	---



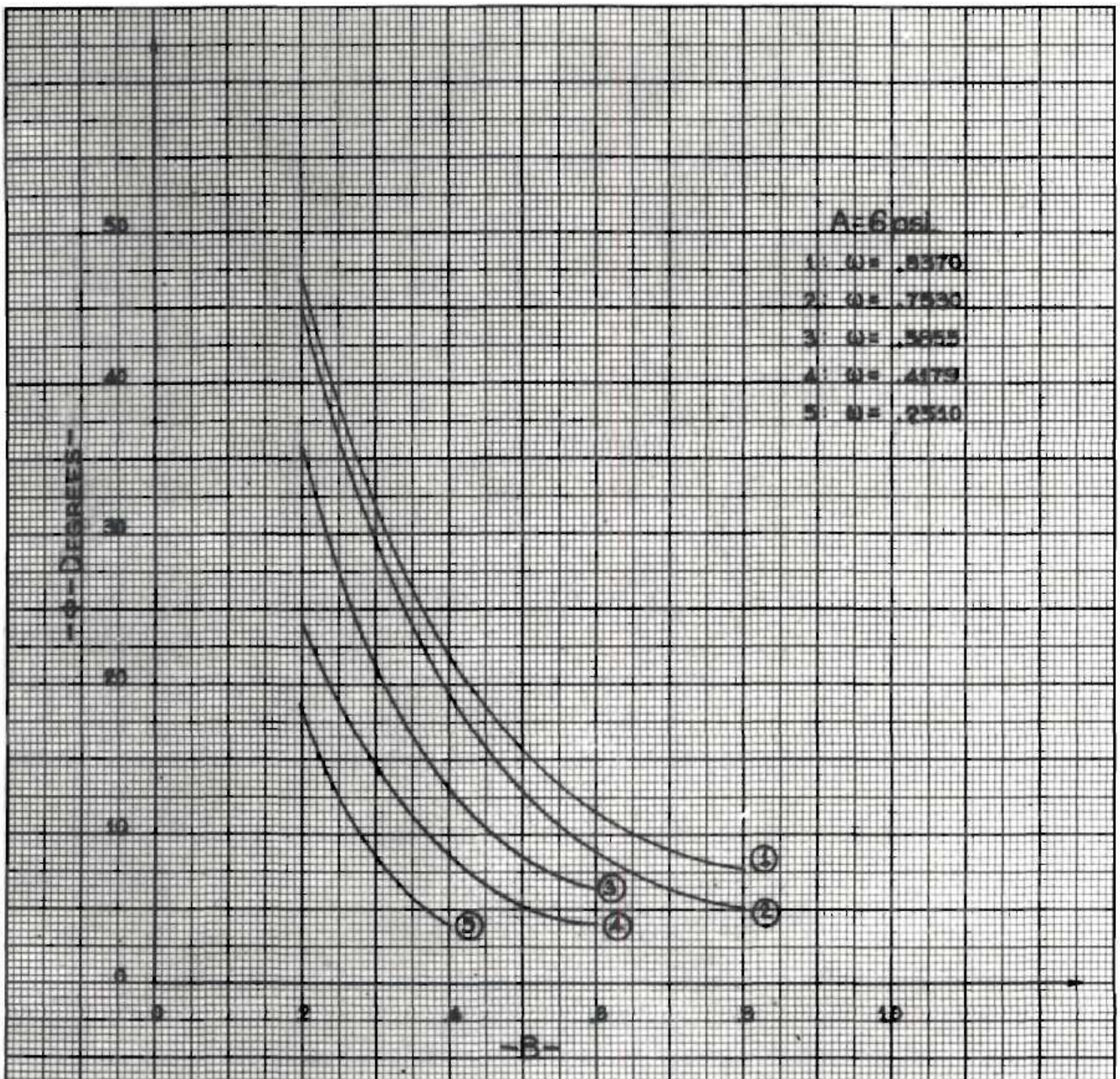
EFFECT OF THE SYSTEM CONSTANT AND THE INPUT FREQUENCY ON THE PHASE SHIFT

FIGURE 5



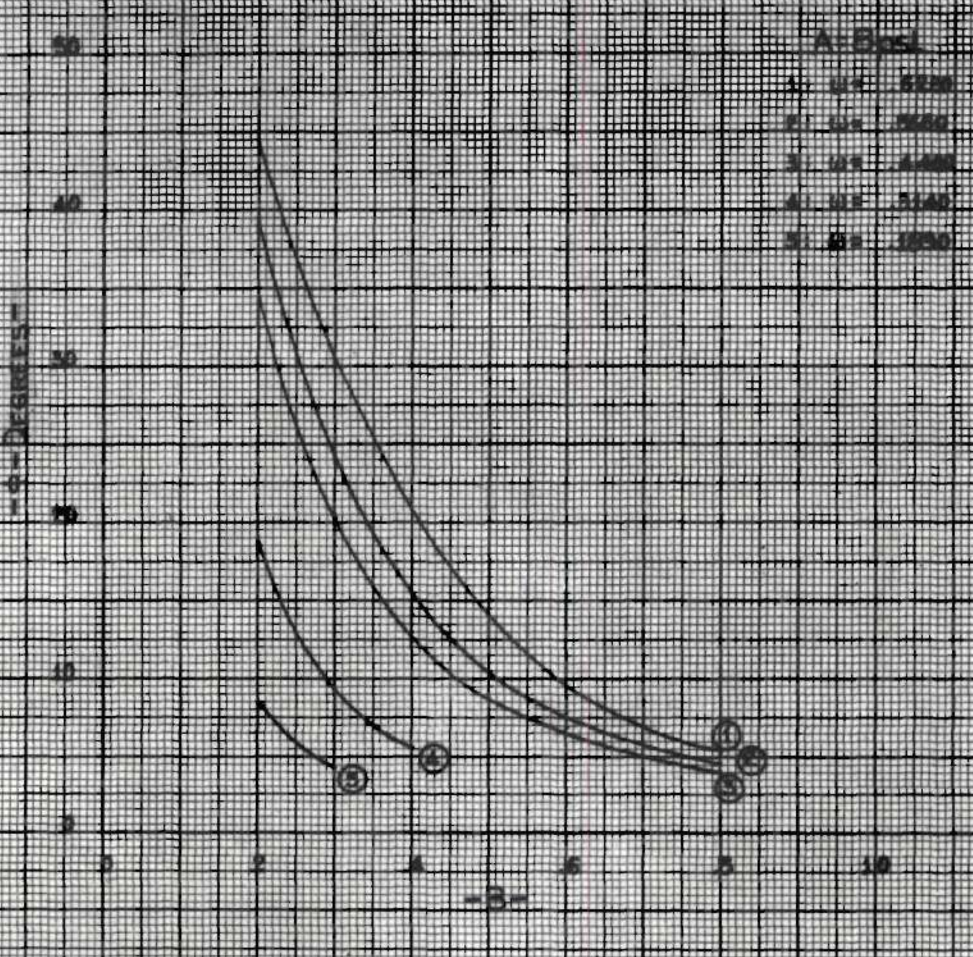
Effect of the System Constant and the Input Frequency on the Phase Shift

Figure 6



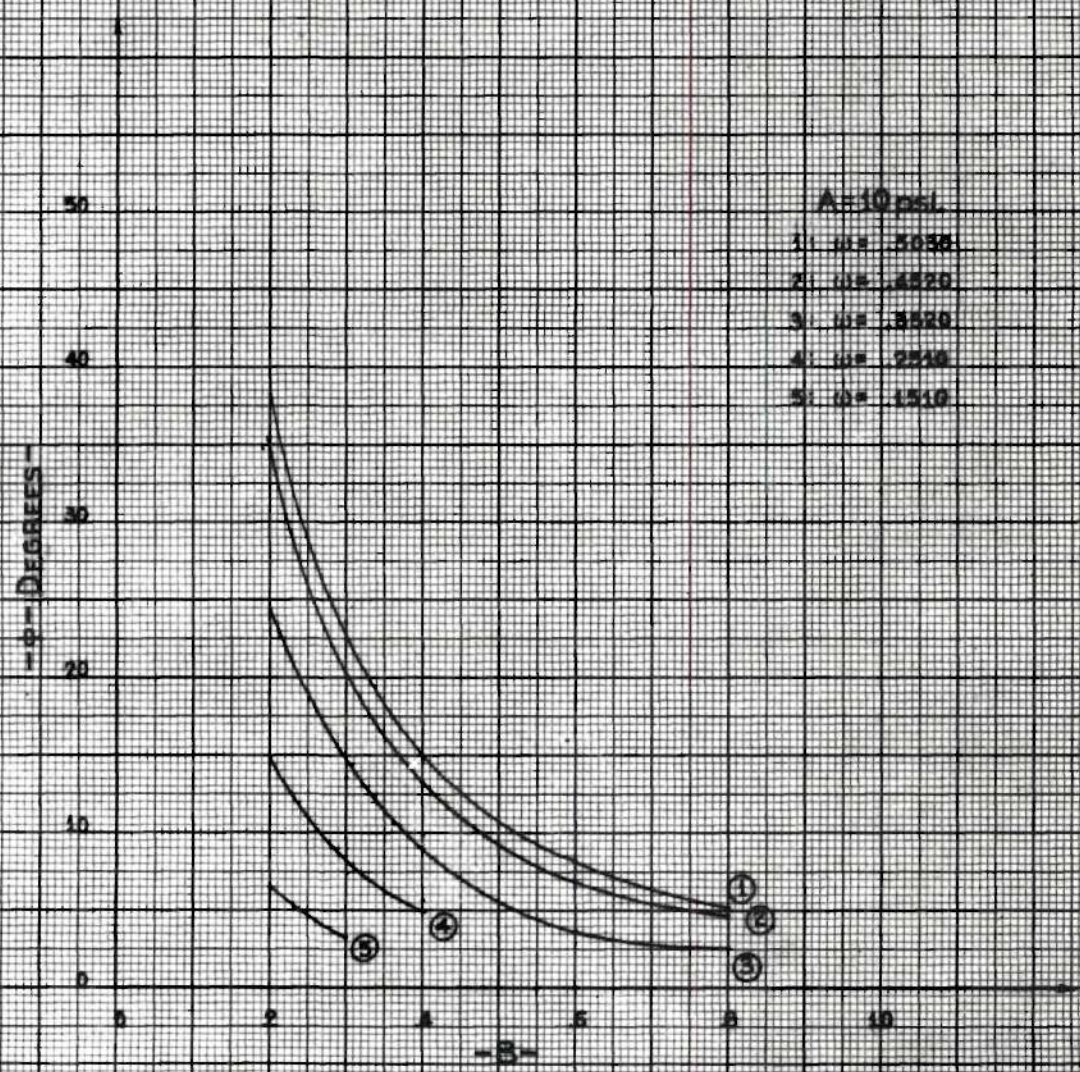
Effect of the System Constant and the Input Frequency on the Phase Shift

Figure 7



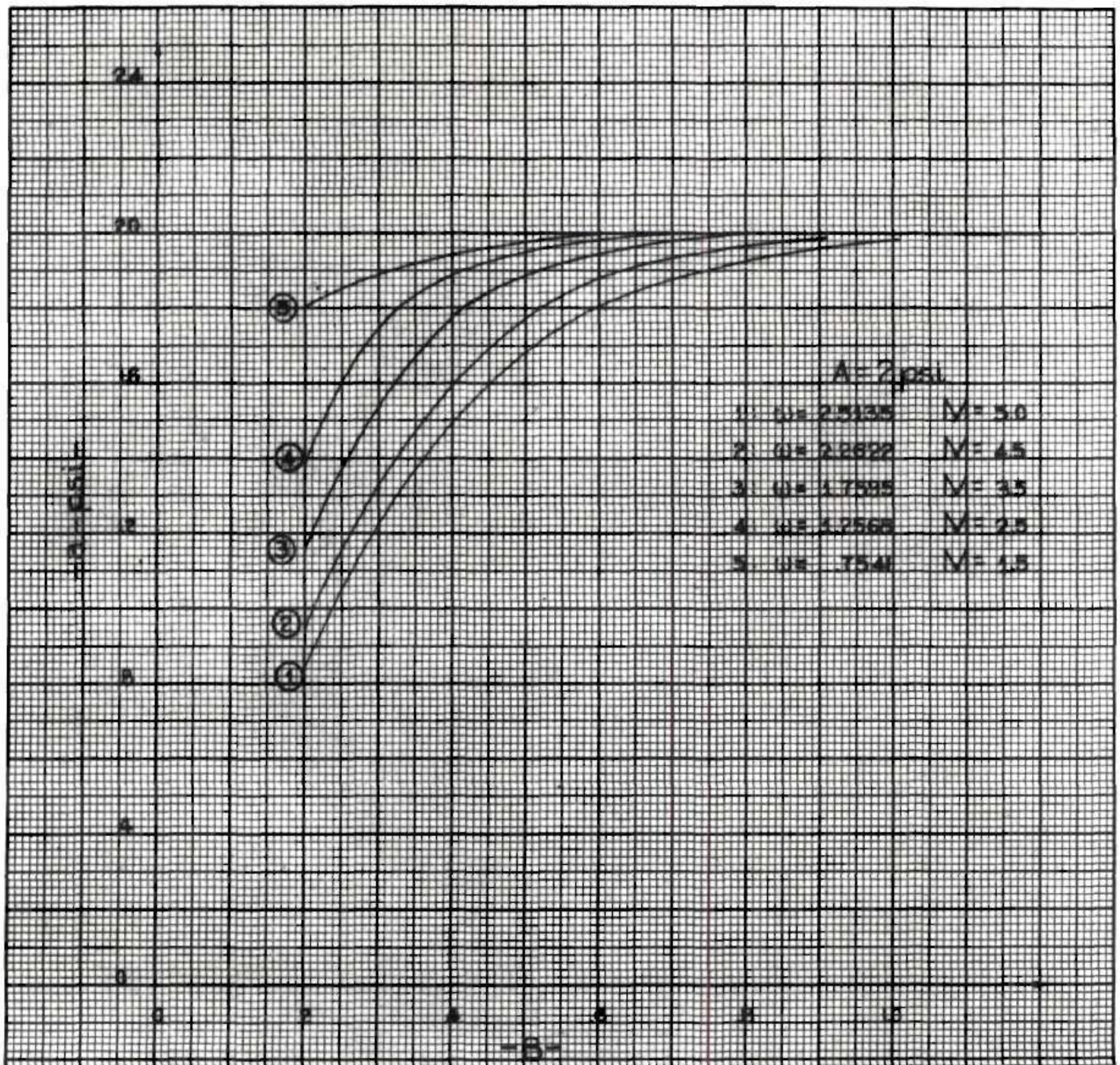
Effect of the System Constant and the Input Frequency on the Phase Shift

Figure 8



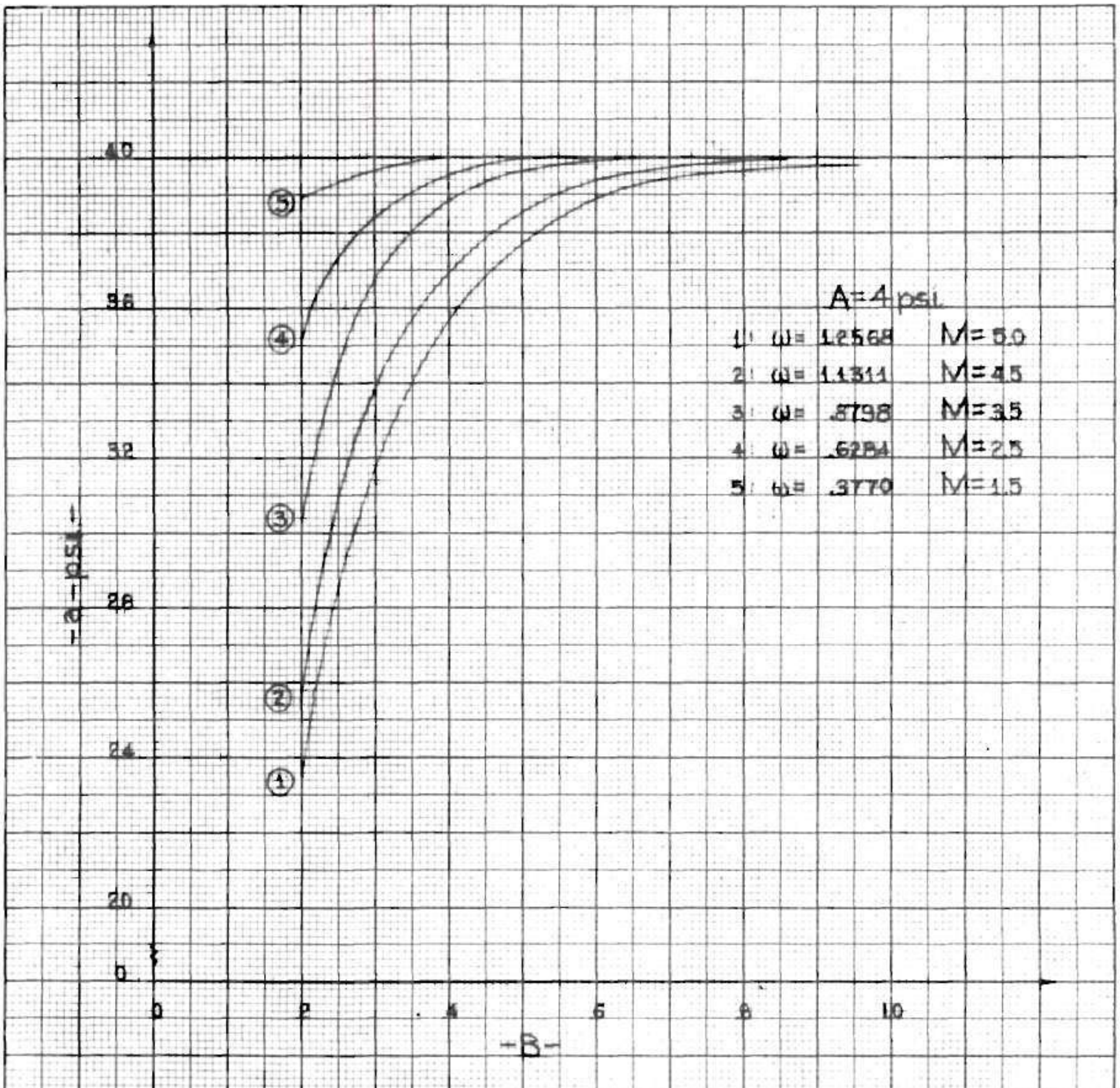
Effect of the System Constant and the Input Frequency on the Phase Shift

Figure 9



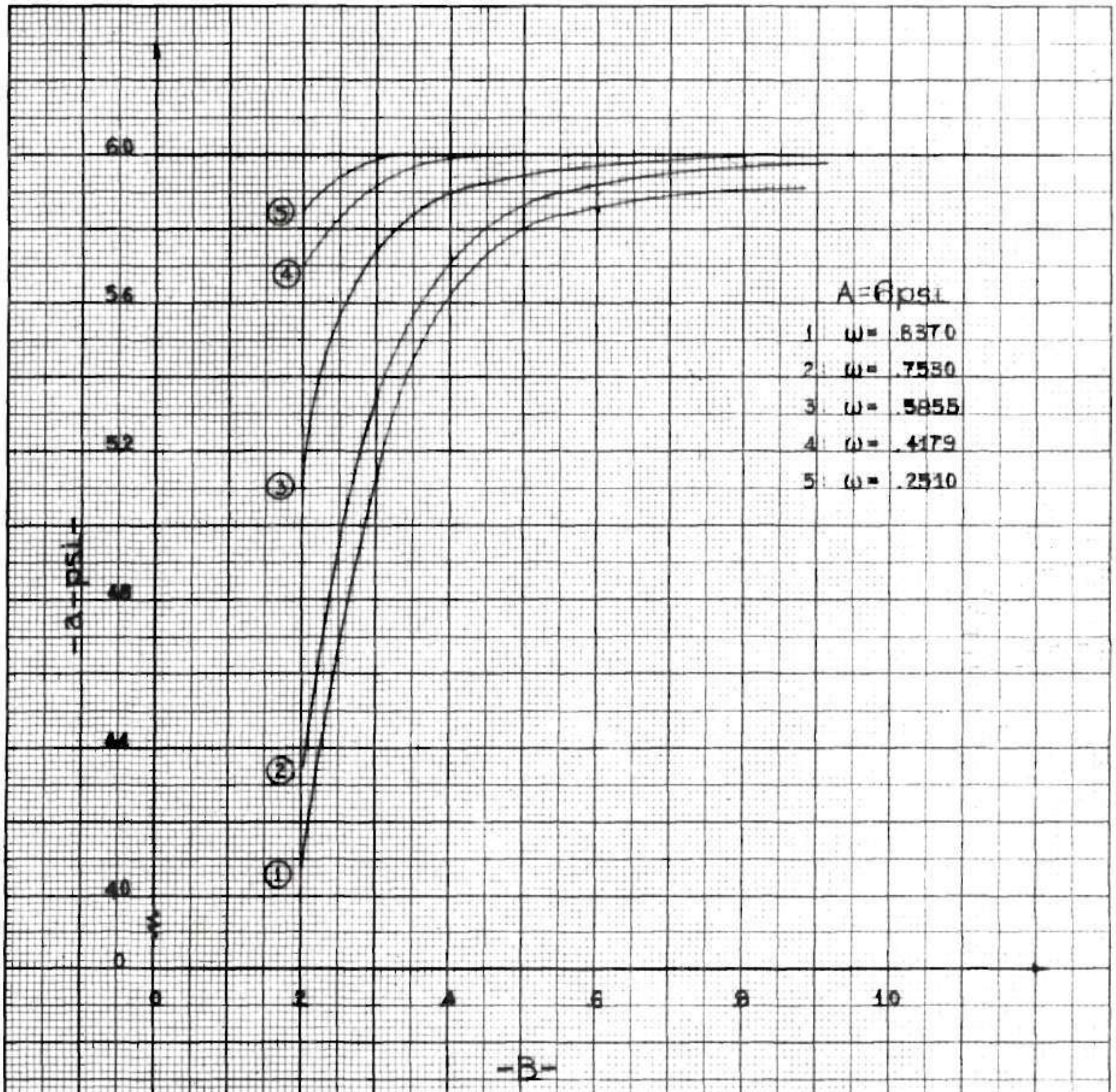
EFFECT OF THE SYSTEM CONSTANT AND THE INPUT FREQUENCY ON THE RESPONSE AMPLITUDE.

FIGURE 10



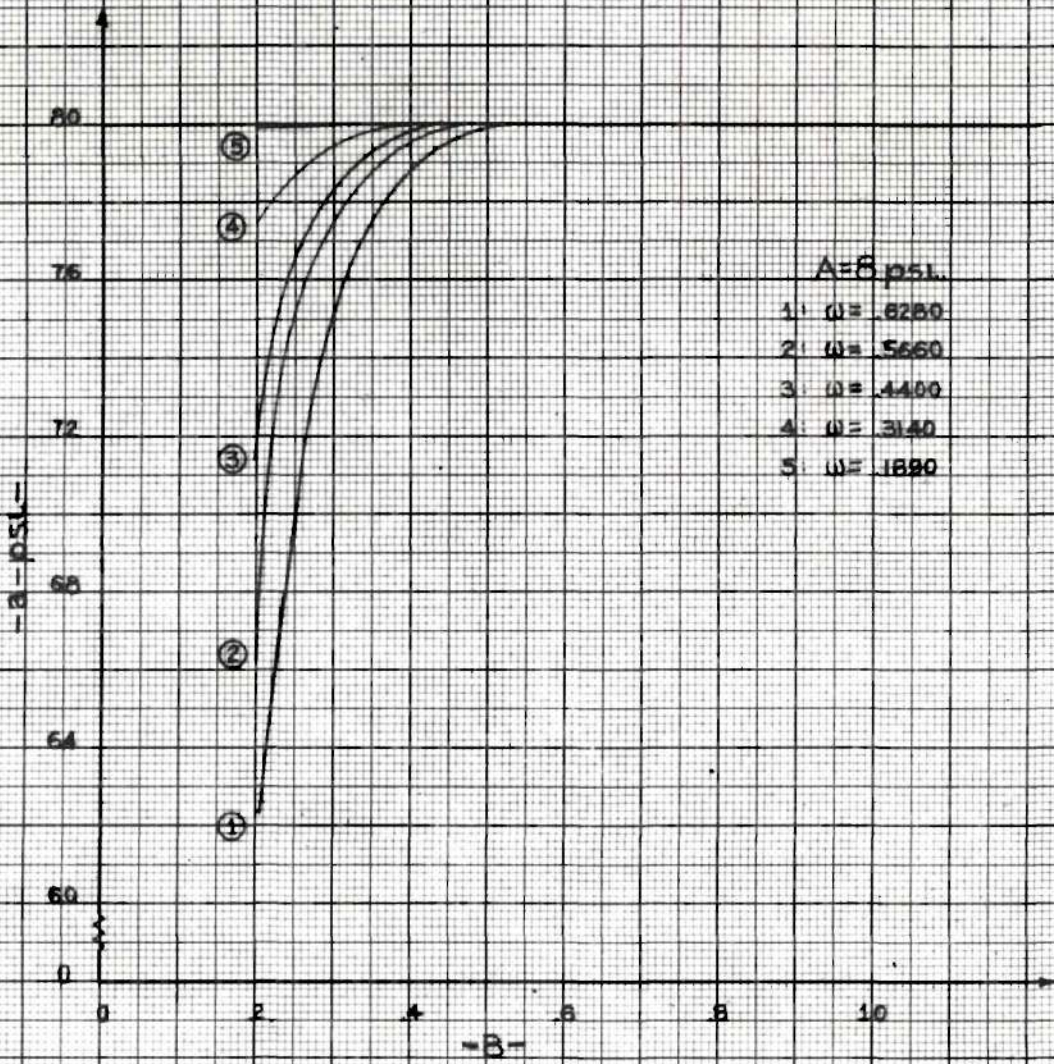
EFFECT OF THE SYSTEM CONSTANT AND THE INPUT FREQUENCY ON THE RESPONSE AMPLITUDE

FIGURE 11



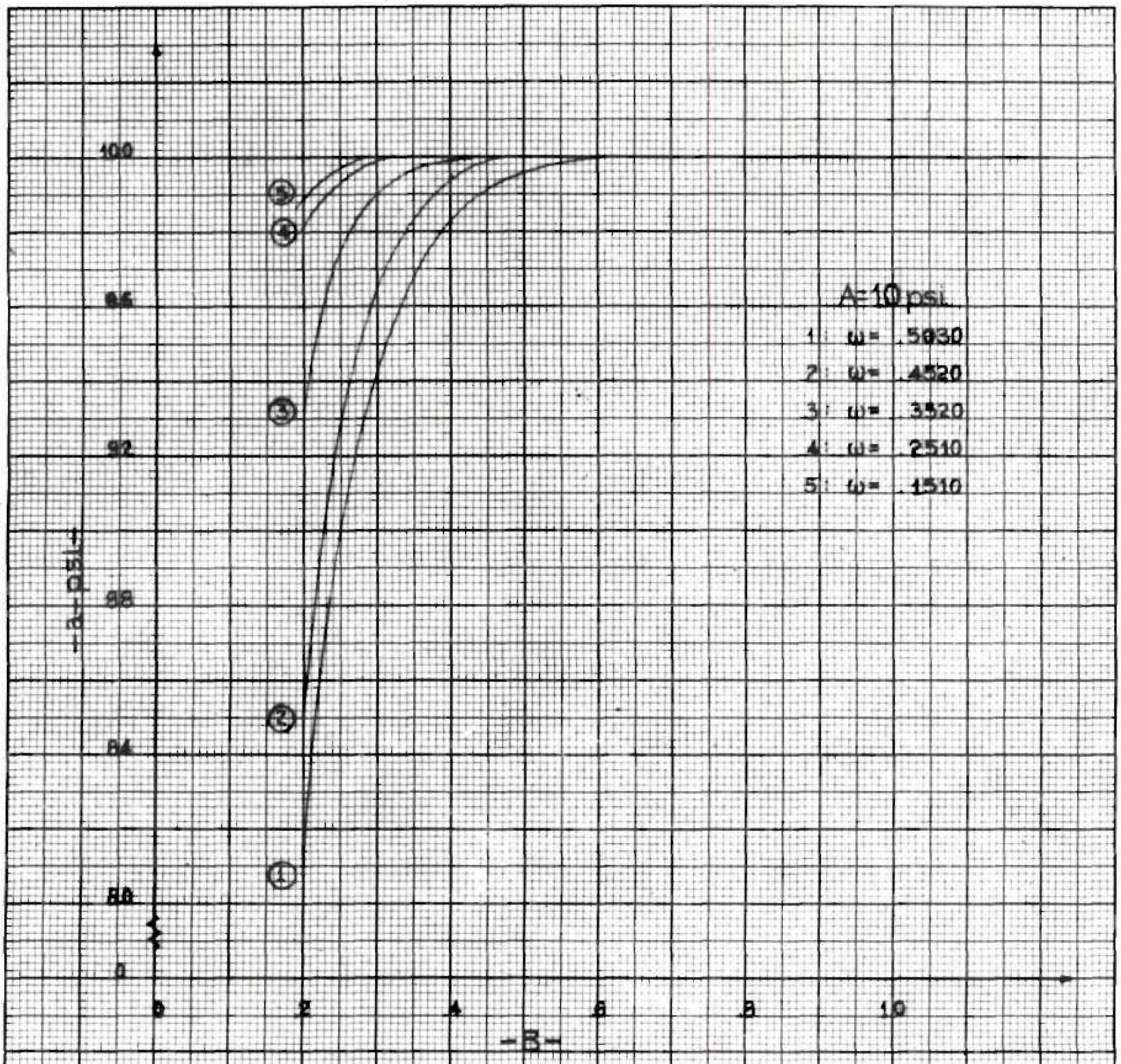
EFFECT OF THE SYSTEM CONSTANT AND THE INPUT FREQUENCY ON THE RESPONSE AMPLITUDE

FIGURE 12



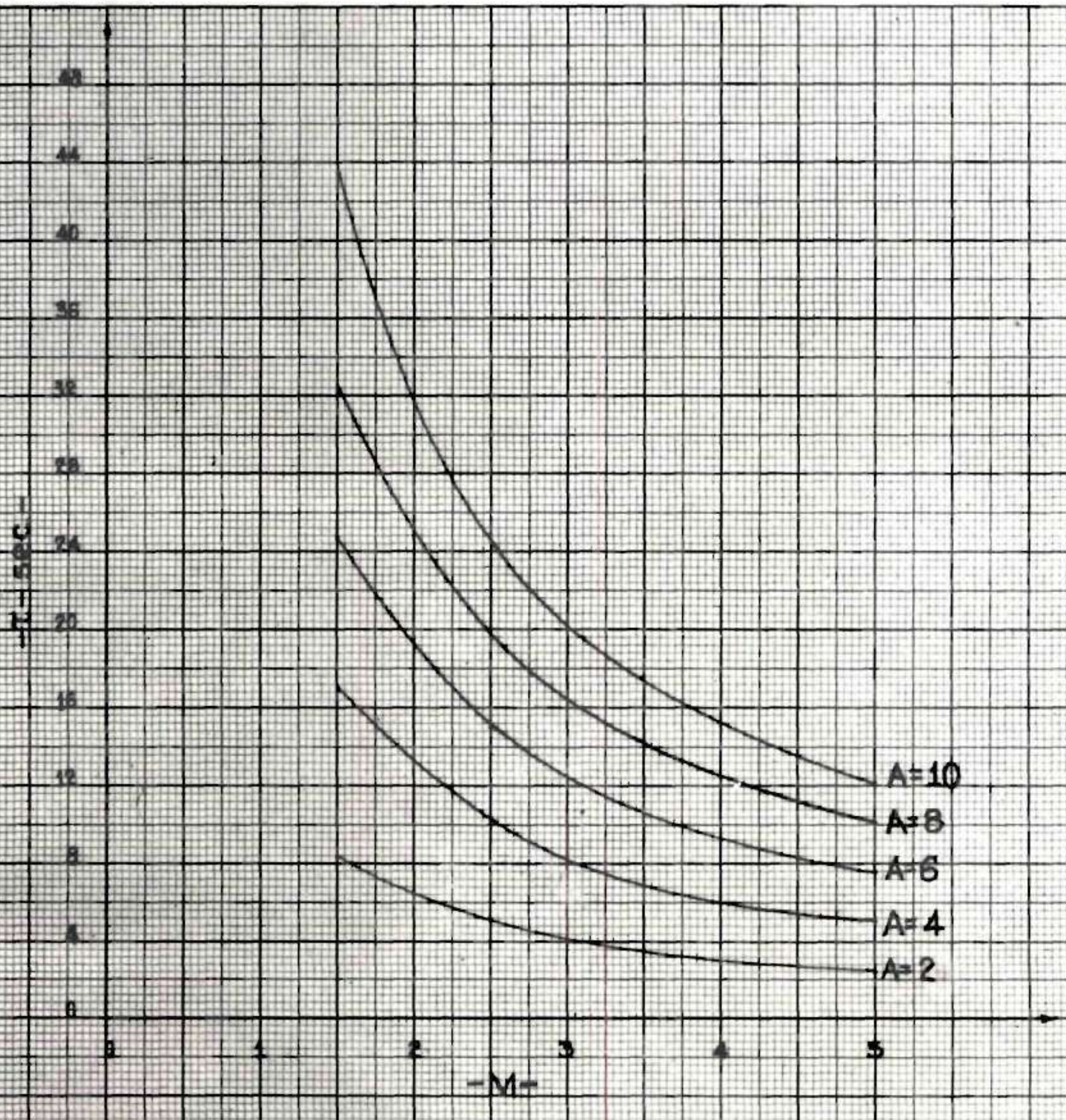
EFFECT OF THE SYSTEM CONSTANT AND THE INPUT FREQUENCY ON THE RESPONSE AMPLITUDE

FIGURE 13



EFFECT OF THE SYSTEM CONSTANT AND THE INPUT FREQUENCY ON THE RESPONSE AMPLITUDE

FIGURE 14



EFFECT OF MACH NUMBER AND INPUT AMPLITUDE ON THE RESPONSE PERIOD (.02 < B < .08)

FIGURE 15

CHAPTER V

CONCLUSIONS

1. The response pressure can be represented by a periodic curve of period slightly different from that of the input function. This periodic curve approximates a cosine curve with some deviation because of the nonlinearity of the system.

2. The response amplitude is always smaller than the input amplitude, depending mainly on the System Constant.

3. There is a phase shift between the input pressure and the response pressure. The smaller the System Constant the larger the phase shift.

4. The solution depends on the altitude.

CHAPTER VI

RECOMMENDATIONS

It must be noticed that equation (7) holds for the subcritical region, therefore the results of this report are true only for $1.0 \geq r \geq .528$. The author recommends that the problem be analysed for the critical region by using the proper differential equation (2).

The System Constant besides being a function of Reynolds number, temperatures, plumbing, and baro sensing systems, is also a function of the pressure ratio to account for compressibility corrections. Note that equation (7) is an incompressible expression. It is recommended that B be expressed as a function of the pressure ratio and the new differential equation be solved for the critical and subcritical regions. This last method would be a good check on the author's present solution.

In this report the problem was solved for an ambient pressure corresponding to 5,000 ft. It would be of interest to solve the problem for other altitudes, or find a method, which will make the solution independent of altitude.

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